Ad soyad:

Numara:

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MATRİSLER TEORİSİ BÜTÜNLEME SINAV SORULARI

1. $A=\begin{bmatrix} 3 & 0 & 1 \\ 2 & -2 & 0 \\ -1 & 5 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 1 & 0 \\ -1 & 2 & 1 \\ 0 & 4 & 1 \end{bmatrix}$, olmak üzere

a) $\det\left(A^2\right)$ **c)** $\det\left(B^{-1}\right)$

2. $A=\begin{bmatrix} 3 & 0 & 3 \\ 0 & 6 & 0 \\ 3 & 0 & 3 \end{bmatrix}$ matrisi verilsin.

a) Karakteristik değerlerini bulunuz. **b)** Karakteristik vektörlerini bulunuz.

3. $\left.\begin{aligned} 2x_1+x_2+x_3&=3 \\ x_1-x_2-x_3&=0 \\ x_1+2x_2+x_3&=0 \end{aligned}\right\}$ lineer denklem sisteminin (varsa) çözümünü bulunuz.

4. $\vec{v_1}=(1,2,0,1)$, $\vec{v_2}=(0,-1,2,1)$, $\vec{v_3}=(2,1,-3,0)$, $\vec{v_4}=(-1,0,1,-2)$

vektörlerinin lineer bağımlılık-bağımsızlık durumunu inceleyiniz.

Süre: 75 dk

BAŞARILAR

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1) $A^2 = \begin{bmatrix} 3 & 0 & 1 \\ 2 & -2 & 0 \\ -1 & 5 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 & 1 \\ 2 & -2 & 0 \\ -1 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 5 & 4 \\ 2 & 4 & 2 \\ 6 & -5 & 0 \end{bmatrix}$

$\det A^2 = 60 - 40 - (96 - 80 + 0) = 4$

veya $\det(A^2) = \det A \det A = 4$

$\det A = -6 + 0 + 10 - (2 + 0 + 0) = 2$

2)

a) $\det(\lambda I - A) = 0$

$$\begin{vmatrix} \lambda-3 & 0 & -3 \\ 0 & \lambda-6 & 0 \\ -3 & 0 & \lambda-3 \end{vmatrix} = 0$$

$(\lambda-3)^2(\lambda-6) - 9(\lambda-6) = 0$

$(\lambda-6)((\lambda-3)^2 - 9) = 0$

$\boxed{\lambda=6}$ ✓ $(\lambda-3)^2 = 9$ $\lambda-3=3$ ✓ $\lambda-3=-3$

$\boxed{\lambda=6}$ ✓ $\boxed{\lambda=0}$

$(-1,0,1)$

$\underline{\lambda=6,}$ $A(\alpha) = \lambda\alpha$ $\alpha = (\alpha_1, \alpha_2, \alpha_3)$

$$\begin{bmatrix} 3 & 0 & 3 \\ 0 & 6 & 0 \\ 3 & 0 & 3 \end{bmatrix}\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 6\alpha_1 \\ 6\alpha_2 \\ 6\alpha_3 \end{bmatrix}$$

$3\alpha_1 + 3\alpha_3 = 6\alpha_1$ $3\alpha_1 = 3\alpha_3$ $\alpha_1 = \alpha_3 = a$

$6\alpha_2 = 6\alpha_2 \Rightarrow \alpha_2 = b$

$3\alpha_1 + 3\alpha_3 = 6\alpha_1 + 6\alpha_3$

$\alpha = (\alpha_1, \alpha_2, \alpha_3) = a(1,0,1) + b(0,1,0)$

karakteristik vektör kar. vekt.

$\lambda=0$ için $\quad A(\alpha)=\lambda\alpha \qquad \alpha=(\alpha_1,\alpha_2,\alpha_3)$

$$\begin{bmatrix} 3 & 0 & 3 \\ 0 & 6 & 0 \\ 3 & 0 & 3 \end{bmatrix}\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3\alpha_1+3\alpha_3=0$$
$$6\alpha_2=0$$
$$3\alpha_1+3\alpha_3=0$$

$\Rightarrow \alpha_1=-\alpha_3=a,\ \alpha_2=0$

$\alpha=(\alpha_1,\alpha_2,\alpha_3)=a(\underbrace{1,0,-1}_{\text{karakteristik vektör}})$

3) $A=\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{vmatrix}=3\neq 0$ olup sistemin tek çözümü vardır.

$$x_1=\frac{\begin{vmatrix} 3 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 1 \end{vmatrix}}{3}=\frac{-3+6}{3}=1 \qquad x_2=\frac{\begin{vmatrix} 2 & 3 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{vmatrix}}{3}=-2$$

$$x_3=\frac{\begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix}}{3}=\frac{6+3}{3}=3$$

$X=(1,-2,3)$

4) v_1,v_2,v_3,v_4 vektörlerinin oluşturduğu kare matris,

$$A=\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & -1 & 2 & 1 \\ 2 & 1 & -3 & 0 \\ -1 & 0 & 1 & -2 \end{bmatrix}$$

matrisi için

$\det A\neq 0 \Rightarrow v_i$ sist. lineer bağımsız,

$\det A=0 \Rightarrow v_i$ lineer bağımlı,

veya elemanter operasyonlar ile,

$$A \overset{E_1}{\sim}\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & -1 & 2 & 1 \\ 0 & -3 & -3 & -2 \\ 0 & 2 & 1 & -1 \end{bmatrix} \overset{E_2}{\sim}\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & -3 & -3 & -2 \\ 0 & 2 & 1 & -1 \end{bmatrix}$$

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$$\sim \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & -9 & -5 \\ 0 & 0 & 5 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & 3-\frac{10}{3} \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 5/9 \\ 0 & 0 & 5 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & \frac{17}{9} \\ 0 & 1 & 0 & \frac{1}{9} \\ 0 & 0 & 1 & 5/9 \\ 0 & 0 & 0 & -\frac{52}{9} \end{bmatrix}$$

$\Rightarrow$ rankA = 4

olup detA $\neq$ 0 dır.

Vi sistemi lineer bağımsızdır